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DISTORTION OF LIQUID CRYSTALS IN THE TWISTED FIELD EFFECT CONFIGURATION

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Abstract The Fréedericksz transition electric field induced distortion of nematic liquid crystals is investigated for the case of the twisted field effect configuration. Experimental capacitance measurements agree well with numerical calculations which allows a reliable determination of the twist elastic constant.

Liquid crystal field effect devices are based on the distortion of the equilibrium orientation by the field. In the simplest case the liquid is aligned homogeneously and the distortion takes place in a single plane.[1] In the twisted nematic cell[2] the orientation of the director is more complicated and the distortion takes place in three dimensions. This problem was first formulated by Leslie[3] who derived the magnetic field threshold condition for alignment parallel to the surface. Berreman[4] solved the same equations numerically in order to calculate the light transmission which is a complex function of the alignment.

To investigate the field induced distortion it is necessary to measure some physical parameter which depends on the orientation of the director. Two simple parameters are birefringence and capacitance both of which depend on an integration of the director orientation. Gerritsma, et al.[5] measured the capacitance of twisted nematic cells as function of the magnetic field. Shtrikman, et al.[6] compared these measurements with a solution of the Leslie equations valid for small distortions. They obtained fair agreement for the distortion threshold, but could not reconcile the data above threshold. We will present similar measurements on electric field induced distortion which are in good agreement with exact calculations.

In calculating the capacitance as function of voltage we start from the differential equations of Leslie[3] modified for electric fields:

$$\cos^2\theta\,(k_{22}\cos^2\theta+k_{33}\sin^2\theta)\frac{d\phi}{dz} = C_1 \tag{1a}$$

$$(k_{11}\cos^2\theta+k_{33}\sin^2\theta)\left(\frac{d\theta}{dz}\right)^2+\cos^2\theta\,(k_{22}\cos^2\theta+k_{33}\sin^2\theta)\left(\frac{d\phi}{dz}\right)^2 - \frac{D_z^2}{\varepsilon_\perp\cos^2\theta+\varepsilon_\parallel\sin^2\theta} = C_2 \tag{1b}$$

C_1 and C_2 are constants of integration to be determined by the boundary conditions. We assume a plane parallel cell configuration with the z-axis perpendicular to the glass plates. The twist angle $\phi(z)$ describes the orientation of the director in the x-y plane, while $\theta(z)$ is the tile angle.

$D_z=(\varepsilon_\perp\cos^2\theta+\varepsilon_\parallel\sin^2\theta)E_z$ is a constant ($\nabla\cdot\underset{\sim}{D}=0$), but E_z is not. (In the magnetic field case both B and H may be assumed to be constant since $\mu\approx 1$.) These equations are solved subject to the boundary conditions of the given alignment at the surface of the liquid crystal. Next, the applied voltage and the capacitance are calculated as function of the distortion and the relationship between them obtained.

If the initial alignment is in the plane of the surface ($\theta=0$) and the directions on the top and bottom of the cell differ by ϕ_0 ($\lesssim\pi/2$) then there is a threshold voltage

$$V_{th} = \pi\sqrt{k_{11}+(\phi_0/\pi)^2(k_{33}-2k_{22})}\Big/\sqrt{\Delta\varepsilon} \tag{2}$$

analogous to the magnetic case. For small distortions above threshold Equations (1a) and (1b) can be solved analytically:[6,7]

$$\frac{\partial c}{\partial v} = 2\frac{\Delta\varepsilon}{\varepsilon_\perp}\left[1+\left(\frac{\phi_0}{\pi}\right)^2\frac{k_{33}-2k_{22}}{k_{11}}\right]\Bigg/\left[\frac{k_{33}}{k_{11}}+\frac{\Delta\varepsilon}{\varepsilon_\perp}+\left(\frac{\phi_0}{\pi}\right)^2\left(-\frac{k_{33}^2}{k_{11}k_{22}}+\frac{k_{33}}{k_{11}}-\frac{k_{22}}{k_{11}}+\frac{\Delta\varepsilon}{\varepsilon_\perp}\frac{k_{33}-2k_{22}}{k_{11}}\right)\right] \tag{3}$$

$$c = \frac{C-C_0}{C_0};\quad v = \frac{V-V_{th}}{V_{th}}$$

where C_0 is the value of capacitance below threshold. In the absence of twist ($\phi_0=0$) the usual results are obtained.[8]

To test the validity of these equations in detail we measured the voltage dependence of capacitance on twisted cells with various alignment configurations. The experimental techniques are very similar to those described in our previous paper.[9] The same liquid crystal mixture of Schiff Base materials was used. It is of positive dielectric anisotropy and the following material parameter values had been determined previously on homogeneously aligned cells.

$$\varepsilon_{\perp} = 5.32, \ \varepsilon_{\parallel} = 8.36, \ k_{11} = 10.8\text{x}10^{-12}\text{N}, \ k_{33} = 10.3\text{x}10^{-12}\text{N}$$

The capacitance was measured as function of a.c. voltage. A typical experimental run on a 90° twist cell is shown in Figure 1. The region near threshold is plotted on an expanded scale in the insert. The solid curve is calculated numerically from Equation (3), using the above parameters and two additional constants

$$k_{22} = (5.7 \pm 1.2)\text{x}10^{-12}\text{N}; \ \theta_0 = 1.5°$$

θ_0 is a small tilt bias arising from imperfect surface alignment. The dashed curve is the one calculated for θ_0=0, the other parameters being unchanged. It shows that the intrinsic threshold, V_{th}=1.965 (Equation 2), is over five percent higher than the measured extrapolated value, due to the small tilt bias. The initial slope, $\partial c/\partial v$=.939 (Equation 3), changes even more with θ_0.

The figure shows the excellent fit between calculations and measurement. The only discrepancy is near the threshold where a slightly larger value of θ_0 would provide a better, but not perfect, fit. However, it is expected that the tilt bias is not uniform throughout the cell which causes some question concerning the application of the theory in the threshold region. This is also confirmed by the fact that the tilt bias varies from cell to cell. In some cases θ_0 was as small as 0.5°, and the measured capacitance values fell close to the dashed curve in the figure. Regardless of the value of θ_0, the experimental points could always be fitted with a curve using the same values of the other parameters. This confirms the validity of the model used here.

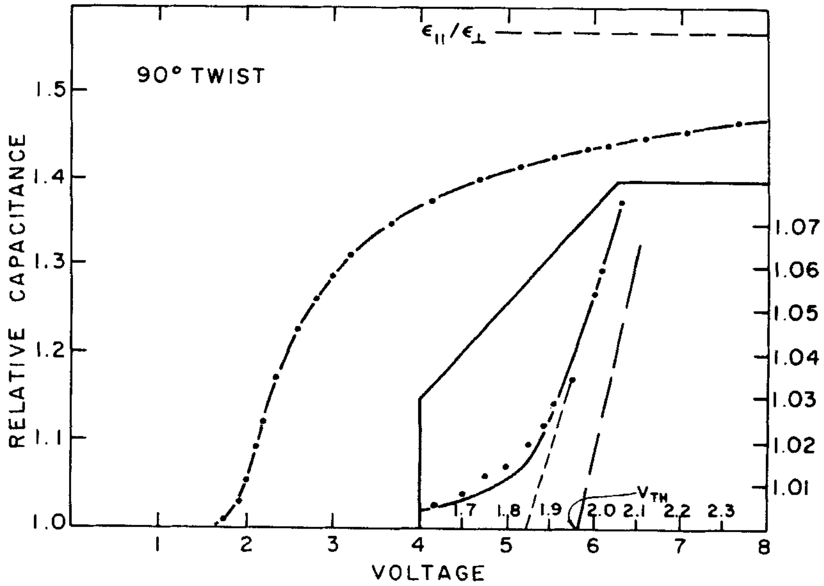

Figure 1. Normalized capacitance as function of voltage for a twisted nematic field effect cell (90° twist). The points are the experimental values, the solid line is calculated using the material parameter values listed in the text. The insert shows the region near threshold on an expanded scale. The short-dashed line is an extrapolation of the experimental data, the long-dashed line is calculated for the case of no initial tilt (θ_0=O).

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